Provable equivalence

Let φ and ψ be formulas of propositional logic. We say that φ and ψ are provably equivalent iff (we write 'iff' for 'if, and only if' in the sequel) the sequents $\varphi \quad \psi$ and $\psi \quad \varphi$ are valid; that is, there is a proof of ψ from φ and another one going the other way around. As seen earlier, we denote that φ and ψ are provably equivalent by $\varphi \quad \psi$.

Note that, by Remark 1.12, we could just as well have defined $\varphi \quad \psi$ to mean that the sequent $(\varphi \to \psi) \wedge (\psi \to \varphi)$ is valid; it defines the same concept.

Examples of provably equivalent formulas are

$\neg(p \wedge q) \quad \neg q \vee$ $\qquad\qquad$ $\neg p \; \neg(p \vee q) \quad \neg q \wedge \neg p$

$p \to q \quad \neg q \to \neg p$ $\qquad\qquad$ $p \to q \quad \neg p \vee q$

$p \wedge q \to p \quad r \vee \neg r$ $\qquad\qquad$ $p \wedge q \to r \quad p \to (q \to r)$.

The reader should prove all of these six equivalences in natural deduction.